# APPLICATION OF ENERGY EQUATIONS IN UNSTEADY CLOSED CONDUIT FLOW

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## ABSTRACT

The application of energy concepts to fluid transients in closed conduits leads to an alternative description of unsteady flow behaviour. In this interpretation, a transient in a pipe system can be viewed as a sequence of energy transformations which moves the system from some initial hydraulic condition to some final state. During this conversion mechanical energy is dissipated and work is done on the fluid.

Depending upon the nature of the flow disturbance, the conversion of energy between its kinetic and internal forms chronicles and characterizes the event. The response of the entire pipe system, or some selected portion of it, can be summarized at each time step by two scalars—(1) the entire kinetic energy of the system, and (2) the system internal energy associated with compressibility effects. Thus, a sequence of two numbers, which may be conveniently plotted as a phase diagram, provides a succinct, quantitative and qualitative description of the unsteady flow phenomenon. This paper compares and contrasts the new energy approach with more traditional interpretations for a number of different pipe systems under a variety of conditions.

Not only does the energy provide an interpretative tool *par excellence*, but the summaries have the potential for defining when and where compressibility effects dominate the system response. In other words, they are useful in determining whether or not the simpler rigid water column theory can be utilized without sacrificing accuracy. Such a characterization has important implications for two related and frequently arising problems, namely that of selecting an appropriate time step for a transient analysis and that of determining the sensitivity of the system to changes in the wavespeed. The usefulness of the energy approach in developing an 'adaptive' procedure to automatically adjust the time step during a transient simulation is illustrated in the paper.

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## Introduction

The scientific study of transient fluid flow has been undertaken since the middle of the nineteenth century. As is true of every other area of engineering research, a great many advances have been made in the accuracy of analysis and the range of applications since then. Although only a few simple problems were approachable by analytical methods and the earlier approximate techniques, a much broader spectrum of transient problems could be solved once graphical methods were developed. More recently, the application of digital computing techniques has resulted in a rapid increase in the range and complexity of problems being studied. The current paper continues this trend by considering the application of energy methods to understanding the dynamics of closed conduit systems during unsteady flow conditions.

When the rate of fluid flow in a closed conduit is changed, large scale conversions of mechanical energy often occur, particularly if the pipeline is carrying water or some other slightly compressible liquid. The various terms that must be accounted for in the analysis include the energy dissipated by fluid friction, the work done at the upstream and downstream ends of the pipe and the kinetic energy carried into and out of the conduit. In addition, if the flow is unsteady, large quantities of energy will be exchanged between kinetic and internal forms. Other forms of energy may play a minor role, such as the acoustic energy sometimes created by transient conditions (water hammer). In essence, the energy approach provides an integrated view of the transient response of a pipeline and therefore provides a simple, efficient and logically consistent way of comparing the behaviour of different systems and different solution techniques.

The energy approach can be easily applied to any continuous system of pipes, continuous in the sense that no two pipes are separated by a boundary condition. Further research in evaluating the work terms across specific types of boundary conditions is required to make the method completely general. The utility of the energy approach is not diminished by this current restriction and this paper explores, by means of examples, a few of its most interesting applications. These include network transient response characterization, flow regime identification and system skeletonization.

## Energy Expressions

The equations of continuity (mass conservation) and momentum are derived in standard references (e.g., Chaudhry, 1987, Wylie and Streeter, 1983). If $x$ is distance along the centerline of the conduit, $t$ is the time and partial derivatives are represented as subscripts, these equations can be written

$$V_t + gH_x + \frac{fV|V|}{2D} = 0 \tag{1}$$

$$H_t + \frac{a^2}{g}V_x = 0 \tag{2}$$

in which, $H = H(x,t)$ = piezometric head, $V = V(x,t)$ = fluid velocity, $D$ = inside pipe diameter, $f$ = Darcy-Weisbach friction factor, $a$ = celerity of the shock wave and $g$ = acceleration due to gravity. To be compatible, $x$ and $V$ must be positive in the same direction.

The continuity and momentum equation (equations 1 and 2) are valid as long as the flow is one dimensional, the conduit properties are constant throughout a given pipe, the "convective" and slope terms are small and the friction force can be approximated by the Darcy-Weisbach formula for steady flow. In addition, it is usually assumed that the friction factor $f$ is either constant or weakly dependent on Reynolds number.

By taking equations 1 and 2 as a basis, it is possible to develop energy expressions (Karney, 1990). In essence, this procedure converts the momentum equation into an expression for kinetic energy by multiplying it by $V$ and intergrating the result over a length of pipe. To obtain an expression for the internal energy, the continuity equation is multiplied by $H$ and also integrated over a length of pipe with constant properties. After intergrating by parts to identify the work terms the result is

$$\frac{\rho A}{2}\left(\frac{g}{a}\right)^2 \frac{d}{dt}\int H^2 dx \;+\; \frac{\rho A}{2}\frac{d}{dt}\int V^2 dx \;+\; \frac{f\rho A}{2D}\int |V|^3 dx$$
$$+\; \rho g A V(L,t)H(L,t) \;-\; \rho g A V(0,t)H(0,t) \;=\; 0. \qquad (3)$$

Each term in this expression has the dimensions of the rate of change of energy and can be written in more compact form as follows:

$$\frac{dU}{dt} + \frac{dT}{dt} + D' + W' = 0 \qquad (4)$$

in which $T$ is the total kinetic energy, $D'$ is the rate of viscous dissipation, and $W'$ is the rate at which work is being done to force fluid into and out of the line. If the only non-zero terms are $D'$ and $W$ the flow is steady. If, in addition, the kinetic energy term is not zero, the flow is unsteady and incompressible. If all four terms are present the flow description is unsteady and compressible.

The meaning of the latter three terms in equation 4 needs no further explanation. The internal energy $U$, however, may not be a familiar concept. Although the changes associated with this quantity are dynamic phenomena, one can easily get a physical sense of the relationship between pressure head changes and the storage of fluid within the pipe from the following quasistatic analogy.

Figure 1 shows a pipe containing water which is closed at one end and has a piston at the other end. Initially, the piston applies just enough force to keep the pipe full of fluid with no static pressure head. If a force is applied to the piston such that the water column is very slowly compressed from its original length of 1001 m to a final length of 1000 m, then, due to the uniform change in mass density of the fluid, the water will experience an increase in pressure. How large the change in pressure actually will be depends upon the elastic properties of the fluid and the pipe wall material since some of the displaced mass is stored by compressing the water (a change in density) and some is accomodated by radial and axial expansion of the pipe material itself. Typical values for the magnitude of the pressure change are indicated in the bottom part of the figure for various degrees of pipe elasticity.

Transients occur whenever rapid local changes cause the fluid to accelerate or decelerate. The 'mass imbalance' thus created in the affected region of pipe is analogous to the piston motion just described and produces changes in pressure, that is, internal energy.

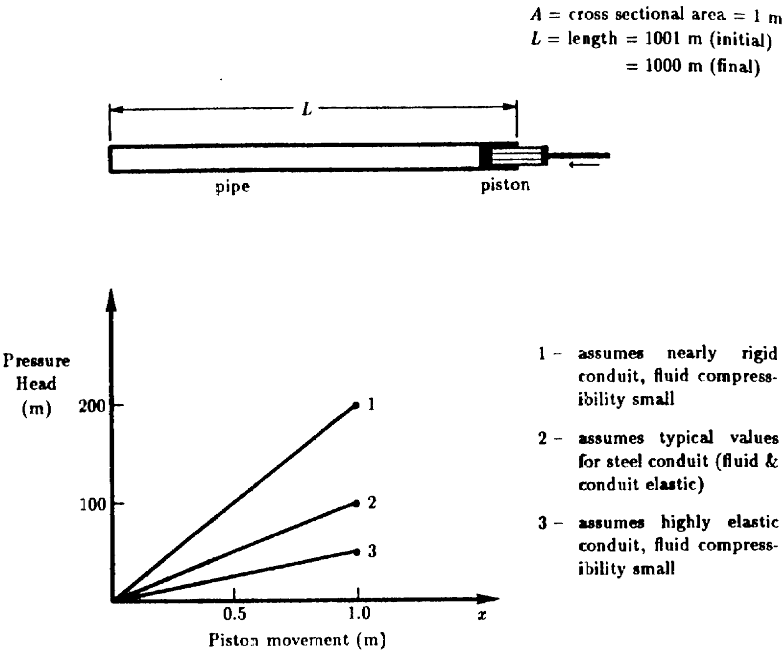


**Figure 1:** Fluid compressibility and pressure head in an isolated pipe.

## Extension to Complex Pipe Systems

In a general pipeline system, equation 3 must be applied to each pipe in turn and all the individual contributions from each pipe summed in order to compute the appropriate energy expression for the entire system. Of course, for a large and complex system such a summation of the individual energy terms may not be the most helpful way of representing the system. Too many important variations within the system may be lost in a single energy term and it may be more useful to consider and compute the energy change in smaller sections of the larger system. A decision as to what level of aggregation will be most useful will depend on the nature of the system and the purpose of the energy analysis.

While equation 3 only applies for a given section of pipe with constant properties, there are several special cases where the equation also holds for the entire conduit. Consider, for example, a series pipeline with a sequence of discrete pipe property changes along its length. If the physical junction between the different pipe sections can be approximated as frictionless, there will be negligible head loss at the junction (in practice, this approximation is frequently made). In this case, two compatibility relations will hold: the head and the discharge at the abutting ends of the pipes will be equal at all times. This boundary condition will then ensure that the work expression at the downstream end of one pipe will be equal in magnitude and opposite in sign from the work term at the upstream end of the next pipe. In this way, all the 'internal work' terms will cancel in pairs. Actually, it can be shown that such a cancellation will occur whenever a connection is made between any number of pipes as long as junction losses are negligible and no flow enters or leaves the pipe system at this location. The energy approach is particularly useful in these applications since a *net* work interaction (and, hence, net change in state for the pipeline) can only occur at the most upstream and downstream ends of the pipeline.

The situation becomes complicated when external flows do occur or when in-line boundary conditions (such as booster pumps) separate groups of pipes. More research is needed to evaluate the specific forms of the work interaction for various types of boundary conditions. Fortunately, the utility of the energy method is not impaired depsite the current lack of appropriate mathematical formulations for nontrivial work terms. In fact, the kinetic and internal forms of energy—which are readily available—provide a wealth of information about the nature of the flow conditions in even the most complex pipe systems.

## Energy Phase Diagrams

It was suggested in the previous section that the response of a pipe, or group of pipes, can be summarized at each time step by evaluating only the internal and kinetic energy terms of equation 4. Doing so produces a set of two scalar values which represents the partition of these two important forms of energy over the period of simulation. When these pairs of values are plotted on orthogonal axes they mark out a path which traces the sequence of energy transformations that took place during the transient event. In this paper, such a plot is called an 'energy phase diagram'. As a practical aside, the internal energy $U$ is actually calculated as

$$U = \frac{\rho A}{2}\left(\frac{g}{a}\right)^2 \frac{d}{dt}\int (H - H_0)^2\, dx \tag{5}$$

so as to remove the effect of an arbitrary datum. This means that $U$ is always zero for the initial condition.

During a transient event there is a dynamic conversion of energy between its kinetic and internal forms. In the absence of any work interactions or frictional dissipation, the conversions would be perfect and the sum of $T$ and $U$ would remain constant over time. This implies that, for fully compressible flow situations, the internal and kinetic energy changes are of the same order of magnitude. This is the reason why these two terms alone can so efficiently characterize the nature of the flow response.

## Energy Transformations in Pipelines

Equation 4 provides a summary of the complete transient response of a pipeline. The focus in the energy approach is no longer the traditional problem of determining what is happening at a particular location in the pipeline. Rather, the central issue is this—to understand how and why the pipeline as a whole responds as it does to transient conditions. To be sure, the question of what happens at a point is important and cannot be entirely forgotten. However, in a system with rapidly propagating and interacting waves, the phenomena at a point cannot be understood in isolation. What happens at a particular location will be better understood and interpreted once the response of the whole system can be explained and summarized in a convenient fashion. This insight is what makes the energy approach powerful.

### Simple Network Example

To illustrate the concise and comprehensive nature of the information provided by the energy method, consider the simple network depicted in Figure 2. This system has seven pipes and seven nodes. The boundary conditions include two constant head reservoirs (with control valves), a floating surge tank with connector, two constant nodal demands, a pressure relief valve (set to open if the piezometric head exceeds 210 m) and a control valve at Node 7 discharging to the atmosphere.

The initial steady state conditions for the system are summarized in Tables 1 and 2. Initially, the control valve at Node 7 has a relative valve opening (or tau value) of 0.6. A transient is induced by closing the valve in 10 seconds to a tau value of 0.2. Three energy subsystems are defined as

**Path 1:** Pipe 1, Pipe 6 and Pipe 7 (P1, P6, P7)
**Path 2:** Pipe 2 and Pipe 3 (P2, P3), and
**Path 3:** Pipe 4 and Pipe 5 (P4, P5).

Note that Path 1 contains the greatest initial kinetic energy and Path 3 the least. Path 2 is dominated by head controlling boundary conditions while the majority of devices on Path 1 are of a flow controlling nature. The role of Path 3 is not immediately clear from these types of considerations. [Note: All flow simulations were performed with TRANSAM, proprietary software of HydraTek Associates, Toronto, Canada.]

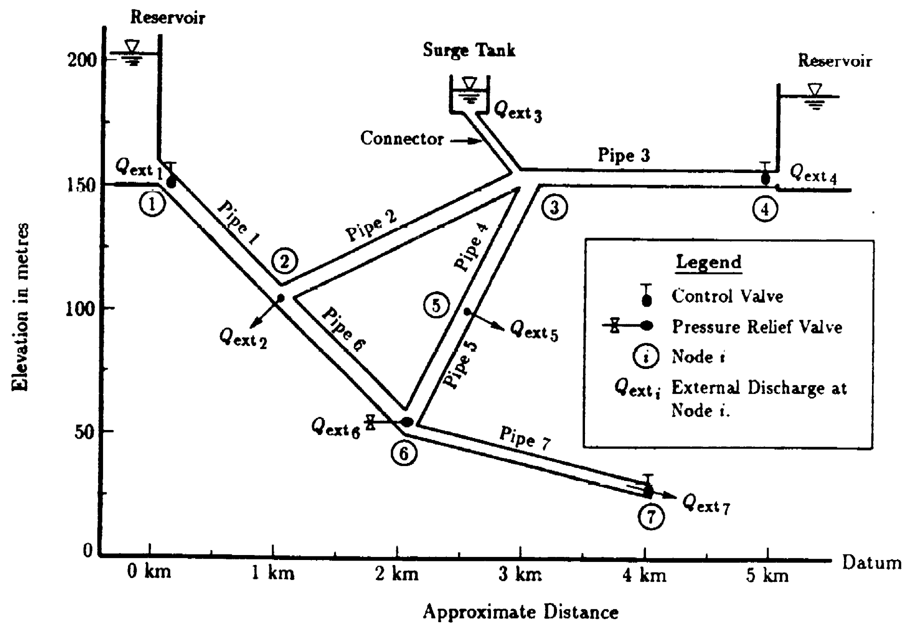


**Figure 2: Definition diagram of a simple network**

**Table 1:** Nodal Steady State Data for Simple Network

| Node Number | Elevation (metres) | HGL Elevation (metres) | $Q_{ext_i}$ ($m^3/s$) | Device Description |
|---|---|---|---|---|
| 1 | 150 | 200.0 | −6.211 | Constant Head Reservoir |
| 2 | 100 | 195.0 | +2.000 | Constant Demand |
| 3 | 150 | 188.8 | +0.000 | Surge Tank |
| 4 | 150 | 175.0 | +1.183 | Constant Head Reservoir |
| 5 | 100 | 183.4 | +1.000 | Constant Demand |
| 6 | 50 | 187.9 | +0.000 | Pressure Relief Valve |
| 7 | 25 | 151.9 | +2.028 | Control Valve to Atmosphere |

**Table 2:** Pipe Physical Data and Initial Flows for Simple Network

| Pipe Number | From Node | To Node | Initial Flow ($m^3/s$) | Length (metres) | Diameter (metres) | Wavespeed (metres/s) | D'Arcy friction |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 6.212 | 1001.2 | 1.500 | 996.3 | 0.012 |
| 2 | 2 | 3 | 1.708 | 2000.0 | 1.000 | 995.3 | 0.013 |
| 3 | 3 | 4 | 1.183 | 2000.0 | 0.750 | 995.0 | 0.014 |
| 4 | 3 | 5 | 0.524 | 502.5 | 0.500 | 1000.0 | 0.015 |
| 5 | 6 | 5 | 0.476 | 502.5 | 0.500 | 1000.0 | 0.015 |
| 6 | 2 | 6 | 2.503 | 1001.2 | 1.000 | 996.3 | 0.014 |
| 7 | 6 | 7 | 2.028 | 2000.2 | 0.750 | 995.1 | 0.013 |

Before looking at the results of the energy analysis, it is instructive to view a more conventional representation of a transient event. Figure 3 shows a composite, three dimensional depiction of the water hammer phenomenon. The figure shows the distribution of piezometric heads in time and space for Path 1, Path 2 and Path 3. The spatial arrangement of the pipe subsystems is not physical but is still convenient for comparing the nature and magnitude of the response in each contiguous group of pipes.

As anticipated, Path 1 experiences the greatest head variation since that is where the flow is undergoing the most significant change. Path 3 shows moderated pressure waves while the least effect is observed in Path 2, an intuitive result since Path 2 contains the surge tank whose function it is to control head fluctuations. Thus, if the importance of compressibilty effects may be inferred from the magnitude of the pressure variations, then the apparent (but incorrect) interpretation is that internal energy changes are important for Path 1, less important for Path 3 and not very important for Path 2.

Next, examine Figure 4, which is an energy phase diagram (actually three phase plots) depicting the transient event whose pressure distribution was just discussed. It is glaringly obvious from the figure that the energy transformations in Path 1 completely dominate the system response. The energy terms for Path 2 are dwarfed by comparison and the response of Path 3 is so miniscule that it could be mistaken for a misplaced axis tick mark. The inserts of the figure indicate that each system, however, has a startlingly complex and well-defined 'structure'.

Considering only Path 1, at time $t = 0$, the internal energy is zero and the total kinetic energy is approximately 24 MJ. As the control valve closes the water is decelerated at the valve

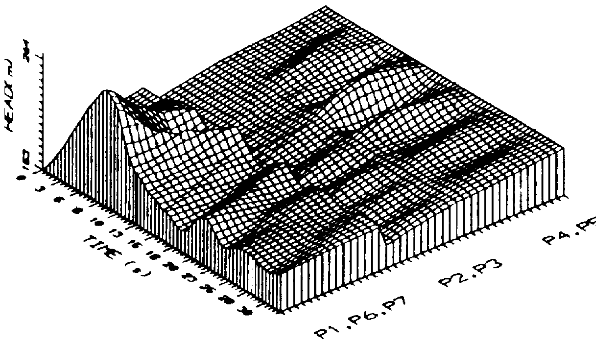


**Figure 3:** Three dimensional representation of simple network transient response.

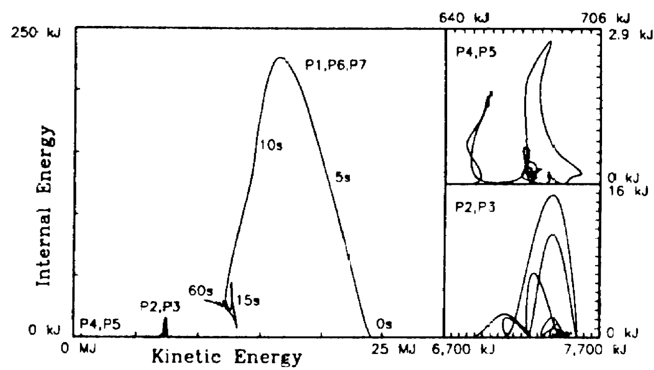


**Figure 4: Left:** Individual subsystem energy phase plots for simple network. **Top Right:** Detail of Pipes 4 and 5. **Bottom Right:** Detail of Pipes 2 and 3. **Note:** Times indicate the number of seconds since the start of the valve closure.

and kinetic energy begins to be converted to internal energy. Over the first four or five seconds of the valve closure there is a virtually linear conversion of kinetic to internal energy. The conversion is much less dramatic than it might otherwise be due to the enormous role of the work terms. The total increase in internal energy is relatively small compared with the change in kinetic energy ( 250 kJ contrasted with nearly 15 MJ respectively) because the release of fluid at several nodes provides a means for dissipating much of the kinetic energy. The transformation process is remarkably stable due to the influence of work interactions and the system rapidly settles down to its new steady state. The conversion process is almost complete 15-20 s after the start of the valve closure. The final steady energy state (at 60 s) shows that the Path 1 kinetic energy has been reduced from about 24 MJ to 10 MJ. The internal energy has increased slightly (relative to the initial steady state heads) because of the reduced friction losses. Note also that the energy phase plots for Path 2 and Path 3 return to a condition near their original energy states.

## Complex Network Example

To further explore the power of the energy approach in characterizing transient behaviour a practical and complex network example is presented in this section. The issues raised here relate to the application of the energy integrals to the evaluation of alternative transient control strategies.

The Bearspaw Northwest Feeder and its associated network form part of the City of Calgary's (Alberta, Canada) large and sophisticated water supply, transmission and distribution system. Figure 5 indicates the complexity of the network which contains numerous pump stations, combination air valves, pressure relief valves and an internal storage reservoir.

The pump station located at the Bearspaw water treatment facility receives water from the Bow River which it pumps to a storage and operating reservoir $5\frac{1}{2}$ km away. The elevation gain is about 135 metres over the length of the 1350 mm reinforced concrete (RC) line which is equipped with several vacuum air valves and one pair of dashpot control air valves. The Bearspaw Treatment Plant pumping station is equipped with surge anticipating valves. In the simulations to follow, only one of the pressure relieving valves from each pair (in the Bearspaw water treatment plant pump station and the dashpot controlled air valves at Station 2750) is assumed to be in service.

The transient scenario most often of concern in systems of this type is the complete loss of power to the pump station while operating at its installed capacity (about 20 MIGD). It is assumed in the simulations that all flow from the Bearspaw treatment plant passes to the Big Hill West reservoir. The other booster pumping stations in the figure are inoperative under these conditions and are treated as dead ends. A major concern on this rising pipeline (see insert to Figure 5) is the occurrence of subatmospheric pressures due to the falling hydraulic grade line subsequent to the failure of the pumps. To prevent these negative pressures from developing, combination air valves are used to admit air to the pipeline at high points; the air forms a cavity and maintains the hydraulic grade at or near the nodal elevation. A significant disadvantage of air valves is that the impact of rejoining water columns can cause severe pressure spikes to be generated. In extreme cases, a 'hybrid' air valve with a pressure relieving action (often called dashpot controlled or slow closing air valves) is sometimes employed. Such valves are installed

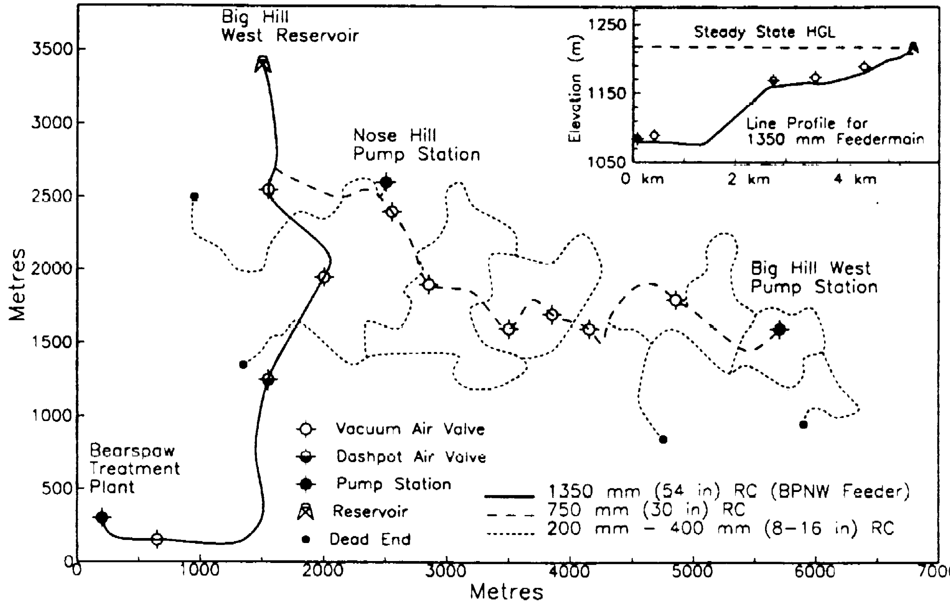


**Figure 5:** Plan view of the Bearspaw Northwest Feeder and associated network.
Insert: Profile sketch of the 1350 mm Bearspaw Northwest Feeder.

at the knee of the Bearspaw Feeder at Station 2750. A generally more effective (and costlier) alternative is a one-way surge tank. Typically, these are small tanks with a free water surface and have a check valve between the tank and the line which opens to admit water to the pipe when the hydraulic grade line falls below the water level in tank. Conversely, the check valve closes upon pressure recovery. A variation on this theme is a surge tank with a pilot controlled valve which opens as before but allows the tank to refill before taking it off-line.

One obvious function of transient analysis is to predict the performance of alternative system configurations under the same set(s) of hydraulic 'loads'. In the current illustration, the hydraulic load is imposed by the pump failure and three pressure control options are to be investigated:

**Case 1:** A slow closing air valve installed at Station 2750

**Case 2:** A one way surge tank installed at Station 2750

**Case 3:** A refilling surge tank installed at Station 2750.

In all cases the Bearspaw treatment pumping station is equipped with a surge anticipating pressure relief valve.

It has not yet been indicated what the performance criterion should be for comparing the three alternatives. Normally, measures like minimum (or maximum) line pressures, either at specific locations or throughout the system would be used to assess the effectiveness of each transient control option. Although these are valid measures of system performance, they provide information of a specific and limited sort. Pressures do not tell the same story as flow variations and even detailed information about each device requires great effort to be comprehensively and conclusively evaluated by the analyst.

On the other hand, the energy approach offers a single, integrated view of the performance of the system as a whole. While it cannot and should not be construed as a substitute for the other types of detailed system information, it enhances the analyst's ability to make general conclusions about the severity of transient effects, the stability and response time of the system, and the role of specific components during transient conditions.

Figure 6 compares a conventional head trace at the pump station with the system energy phase diagrams for the simulated Cases 1–3. Without going into a tedious account of the behaviour of the Bearspaw Feeder, several key features in the figure can be remarked upon. All three responses are much the same over the initial downsurge period (point A in the plots). It is worth noting that the internal energy is increasing because of the $H^2$ in equation 4, that is, whether the fluid column is under tension or compression, the internal energy is always a positive quantity. Between A and B a period of pressure recovery is evident. The horizontal loop between B and C corresponds to the formation and collapse of air cavities in the system. The period of transient decay begins at the point D and continues to the conclusion of the simulation.

Although it is clear from the head traces that the refilling surge tank option offers little improvment over the slow closing air valve alternative, it is impossible to understand on the basis of the pump station heads the reason(s) for this counter-intuitive fact. What is remarkable about the energy phase diagram is that it shows immediately why this is so. The system is not stable with respect to air cavity formation at certain locations in the associated network despite

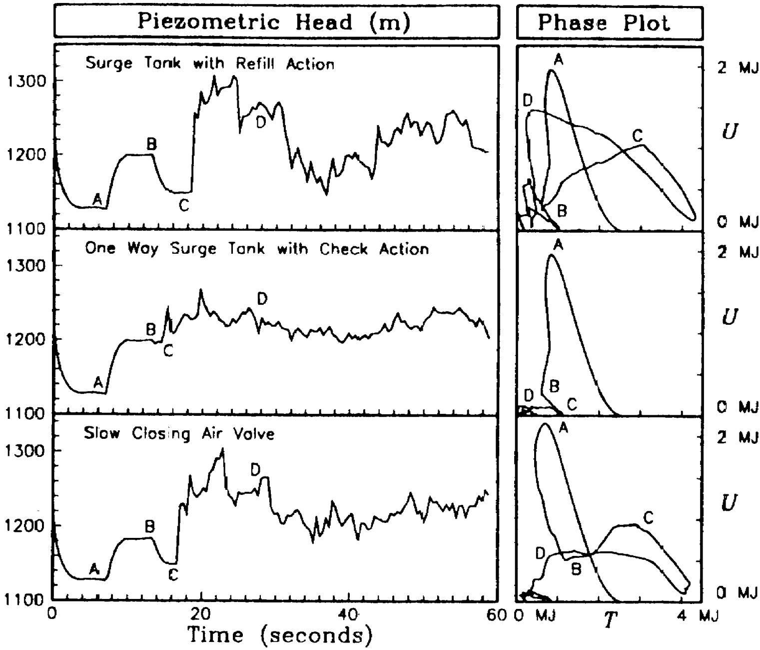


**Figure 6:** **Left**: Simulated piezometric head traces at the Bearspaw Treatment Plant pumping station. **Right:** Energy phase plots for the Bearspaw Northwest Feeder. **Bottom:** Case 1: Slow closing air valve installed at Station 2750. **Centre:** Case 2: One way surge tank with check action installed at Station 2750. **Top:** Surge tank with refilling action installed at Station 2750.

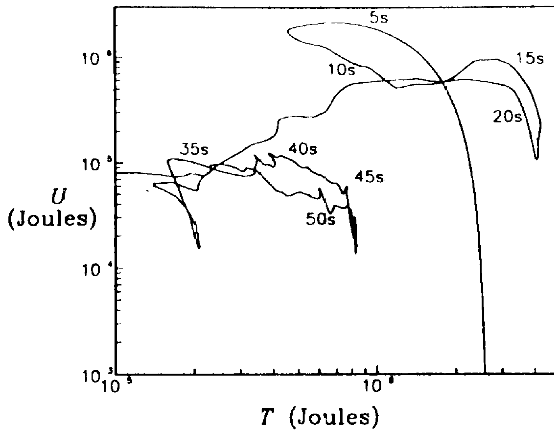


**Figure 7:** Energy phase plot (Bearspaw Northwest Feeder—Case 1) with logarithmic axes.

the fact that the surge tank completely controls the low pressure problem on the Bearspaw Northwest Feeder itself. By contrast, the one way surge tank alternative appears very stable in this regard and is clearly superior to the other two design options.

A final difference between conventional transient response descriptions and those of the energy method is illustrated in Figure 7. The fluctuation in piezometric head appears rather chaotic in the previous figures. Using logarithmic axes to plot the energy phase diagram for Case 1 unexpectedly reveals the self-similarity and repetitive structure of the transient phenomenon during the period of decay. Although the changes in energy span several orders of magnitude the same pattern of events unfolds, slightly modified by work and dissipation effects, each cycle.

## Importance of Compressibility

In many ways, the systems that have been discussed so far are quite typical. Closed conduits frequently carry huge amounts of momentum and kinetic energy. Only when changes in flow rate take place very gradually, such that the mass and energy imbalances in the line are always small, is it possible to smoothly change the flow from one steady condition to another. In such applications it *may* be justified to neglect compressibility effects and use the rigid water column model mentioned earlier in this paper. However, if rapid changes occur, whether caused by standard operating procedures or accidental events, relatively large mass and energy imbalances may arise. The associated pressure pulses are of high magnitude and are capable of bursting or damaging pipelines.

However, one of the most difficult questions to answer is "How rapid is rapid?" That is, how can the analyst know whether or not compressibility effects are important or how much error might be introduced by using the rigid water column model? As the following discussion shows, energy methods provide a general and powerful way of answering such questions.

In essence, when the equilibrium conditions in a conduit change, work must be done on the pipeline. However, if the equilibrium condition is changing more rapidly than can be accommodated through the work terms, energy storage—that is, the transformation of energy between the kinetic and internal forms—becomes progressively more important. Thus, the ratio of the total change in internal energy to the total change in kinetic energy provides a natural index of the importance of compressibility effects. Thus, if $\phi$ is the 'compressibility index' then

$$\phi = \frac{\Delta U_{max}}{\Delta T_{max}}. \tag{6}$$

In the limit as $\Delta U_{max}$ approaches zero, the compressibility index $\phi$ will approach zero. It can be seen from the original definition of $U$ (the first term in equation 3) that the limit of zero is also achieved as the wavespeed $a$ approaches infinity. This observation is in full agreement with the assumption of instantaneous propagation of flow disturbances in the rigid water column model.

The power of the $\phi$-index to characterize a pipeline's response has important implications to transient analysis. In most applications, the analyst does not make only one computer run; rather, many different initial states and device combinations may be investigated to identify which conditions are most severe. For a given series of these numerical experiments, only one run may be required to determine the $\phi$-index in a general way for the whole set. Suppose this is done and the index turns out to be much less than one. This would mean that the pipeline's behaviour is insensitive to adjustments in the value of the wavespeed and that the pipeline can be approximated by the rigid water column model.

It should be emphasized that the conclusions based on the $\phi$-index are independent of location and time in the pipeline. The energy terms used to define the value of $\phi$ relate to the entire pipeline and as such do not depend on where in the pipe or when in the simulation the term is evaluated. As a result, the $\phi$-index has considerable potential for application in adaptive algorithms: that is, the index could be evaluated to determine the influence of compressibility during a particular time step; if compressibility is important, the current time step can be decreased to improve accuracy; on the other hand, if compressibility effects are not important, it might be possible to increase the time step (say by wavespeed adjustments) and obtain an equivalent degree of accuracy at less expense. More work is needed to further refine these ideas.

As a result of the above benefits, the energy relations provide a natural means of quantifying errors in the analysis that might arise because of differences in either the physical representation of the pipe system itself or in mathematical approximations used in modelling the system. Most significantly, the energy relations allow the role of compressibility and energy storage under transient conditions to be quantified—as the compressibility index approaches zero, the rigid water column model becomes a better approximation of the transient behaviour of the system. Such generalizations are a direct outcome of the simplicity and power of the energy methods of transient flow analysis.

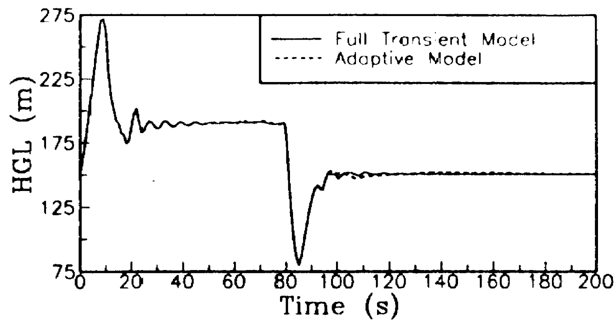


**Figure 8:** Adaptive procedure comparison of simulated piezometric heads at Node 7 for the simple network case.

## Adaptive Model

To briefly illustrate the practical utility of these assertions, the transient model TRANSAM was altered to provide a simple adaptive capability. The kinetic and internal energy changes are monitored over a selected time period and the $\phi$-index calculated. A threshhold value for $\phi$ of 0.015 produced good results with respect to both accuracy and execution time. Using the same simple network shown in Figure 2, a modified valve behaviour (the valve reopens between 80 and 85 seconds) and a longer duration simulation, the adaptive procedure achieved a 40% reduction in the simulation execution time. Figure 8 shows the correspondence between the simulated piezometric head variations at Node 7 for the full transient model and the adaptive model.

The decrease in execution times under the adaptive procedure is accomplished by doubling the time step whenever the threshold $\phi$ value is reached. This approach works because a suitably small value of $\phi$ means that compressibility effects are negligible, that is, the term in the governing equations which contains wavespeed is very small. Because the compressibility term varies inversely with $a$ (see Wylie and Streeter, 1982) decreasing the wavespeed will make this quantity larger. Theory states that as flow becomes fully incompressible the wavespeed tends to infinity. It seems therefore wrong that reducing the wavespeed is consistent with incompressible flow. In fact, it is counter to what we would expect and does introduce error into the affected term. However, provided that $a$ remains much greater than $V$, the error produced is still negligible compared to the magnitude of the other terms in the governing equations. It is this obscure fact which makes the method practical. By doubling the time step, the wavespeed is halved and the value of $U$ increases by a factor of four (see equation 4). Thus, the $\phi$ threshold can be passed more than once and a series of progressive time step doublings can take place. The simulation model uses a 'look ahead' scheme to determine whether or not control actions will occur over the next time interval and the model reverts to the original transient calculations if required.

More efficient use of computer and human resources is one benefit to be realized from adaptive modeling techniques. Another advantageous use is an automatic stopping criterion for transient simulations—once the compressibility effects become negligible the execution is terminated -- relieving the analyst of this responsibility. The possibility for bringing the full power of the dynamic flow equations to bear on extended period, incompressible flow calculations is also an exciting prospect.

# System Specification

One of the most difficult aspects of modeling the unsteady flow behaviour of complex systems is knowing what simplifications in both the data and the model representation are permissible. Which system components are essential if the model is to faithfully reproduce the behaviour of the real pipe system? What level of discretization is needed to produce the desired accuracy? Can simpler, more efficient representations be found for complex or problematic components? The energy method promises to be an excellent tool for making these kinds of modeling decisions. Because of its concise and summarizing nature, alternate specifications of the same system can be compared quickly and comprehensively, allowing the analyst to determine which level of representation best achieves the modeling objectives with the least expenditure of resources. The implications for cost effective analysis are illustrated in the following example.

## Boundary Condition Representation

A 1000 m long steel pipeline carries water from a reservoir to a centre pivot irrigation system. The centre pivot aluminum pipe is 100 m long and has five orifices equally spaced along its length which spray water onto the crops. An in-line control valve for bringing the pivot on and off line is installed at the pivot tap-off. The wavespeeds of the aluminum and steel pipe are assumed to be 1000 m/s. Sufficient head is available at the reservoir to ensure proper operation of the pivot.

To model the pivot rigorously it is necessary to split it into five pipes each 20 m in length and each with an orifice boundary condition at the downstream node. This forces the analysis to adopt a (maximum) reach length of 20 m and at least 50 reaches are required to model the supply pipeline. In addition, the time step is computed as

$$\Delta t = \frac{\Delta x}{a} = \frac{20}{1000} = .02 \text{ seconds.} \qquad (7)$$

Thus, there are a total of 55 pipe reaches in the system and 50 time steps to be solved for each second of real time. On the order of 56 × 50 or roughly 2,800 solution points per second of simulated time must be performed by the model. Is there a more efficient way to specify the centre pivot boundary condition which preserves the essential character and accuracy of the predicted transient response? One possibility is to let the entire centre pivot device be modeled as a single 100 m aluminum pipe with all of the flow discharged from a terminal orifice. This permits a (maximum) reach length of 100 metres to be used and increases the time step to 0.1 seconds, roughly 110 solution points per second of simulated time.

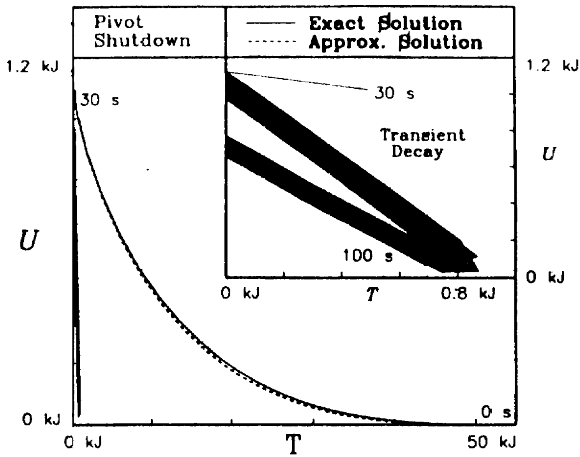


**Figure 9:** Comparison of simulated energy response for the 1000 metre pipeline under different pivot representations. **Insert:** Detail of phase plot decay to static conditions following closure of the in-line control valve.

Figure 9 shows an energy phase diagram for the transient produced when the pivot is taken off line. The control valve has a linear closure over a period of 30 seconds in the example and the response has been simulated over a 100 second interval. The results clearly indicate that the error in the approximate 'model' of the centre pivot is marginal. The difference in execution times however is not. The execution time was reduced by a factor of 4.5 for the single orifice representation. The computer time needed to run the latter system is primarily due to input and output and the number of CPU seconds actually spent performing computations would be far less than this figure would imply.

Although this example is somewhat trivial, consider the computer resources required for a larger system containing several kilometres of pipelines and possibly ten or twenty pivot locations. Such realistic cases could strain the limits of most transient models. Making even such an apparently trivial simplification as that described here could mean the difference between a system which is impossible to model and one which can be not only analysed, but investigated with equal confidence in the model results.

## Summary

The traditional approach to the analysis of transient conditions in pipeline systems makes use of the continuity and momentum equations to produce a detailed picture of pressure and flow variations at discrete locations in the system. Although a wealth of specific, local information is produced by conventional models it may be difficult for the analyst to piece together a composite picture of the entire system response. The new energy equations afford the transient investigator an integrated and insightful vista into the global behaviour of pipe systems undergoing rapid changes in flow conditions.

But the power of the energy method goes further than simply offering another interpretive tool—it also gives the analsyt the considerable advantage of knowing when and where a simpler unsteady flow description can be successfully applied. The benefits of this adaptive approach are the ability to run simulations for either longer periods, more quickly, or both. It means that with a single mathematical formulation, and within a single model execution, it is now practical to analyse unsteady compressible or incompressible flows as dictated by the actual system conditions themselves. This has important implications to fulfilling the desire among many waterworks utilities for the development of a single, integrated modeling, control and data base system.

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