Transient Analysis with Time-Decoupled Pumping Station

By David H. Axworthy,[1] Associate Member, ASCE, and Bryan W. Karney,[2] Member, ASCE

Abstract: An explicit numerical approximation of the inertial equation governing pump speed changes is shown to have many computational advantages over the conventional implicit approach when modeling complex pumping stations. The pump boundary condition is numerically decoupled from the transient time step by solving the first-order differential torque equation explicitly. A step-by-step finite difference method is used to integrate the torque equation, but the energy equation is solved separately by Newton's method. The explicit approach is demonstrated on a simple forcemain and shows good agreement with the conventional implicit approach. Applications show the ease with which complex and variable speed pumping arrangements are efficiently modeled using the explicit approach. In particular, the explicit approach does not require: (1) contraction of the system of equations as a decelerating pump is eliminated upon pump check valve closure; (2) expansion of the system of equations as an accelerating pump comes up to speed; or (3) dedicated code for combinations of operating, failing, or speed-changing pumps.

INTRODUCTION

This paper reviews the explicit formulation of the pump boundary condition first presented by Streeter (1969) and contrasts it with the conventional implicit approach. In general, the explicit approach is subject to the same inaccuracies and uncertainties as the implicit approach. Therefore, the explicit approach realizes the same advantages and disadvantages with respect to comparison of computed and measured results. Rather the benefits, although significant, are purely computational and until now have largely been ignored. Due to its simplified bookkeeping, relating to soft starting and stopping of pumps, the explicit approach is particularly helpful when modeling complex and variable speed pumping arrangements. To set these developments in context, a brief review of the pump boundary condition mathematical formulation, as it applies to the method of characteristics (MOC), is appropriate.

DEVELOPMENT OF PUMP BOUNDARY CONDITION

The energy equation requires that the sum of the suction head H_S and the dynamic head of the pump H_P equal the sum of the head losses at the pump valves H_υ and the discharge head H_D. In symbols, the energy equation is

$$F_1 = H_S + H_P - H_\upsilon - H_D = 0 \tag{1}$$

in which H_P = dynamic head of pump and is a function of relative pump speed $\alpha = N/N_R$ and relative pump discharge $\upsilon = Q/Q_R$; suction and discharge heads H_S and H_D are defined by the MOC characteristic equations (see Karney and McInnis [1992]); Q = pump discharge; Q_R = rated pump discharge; N_R = rated pump rotational speed; and N = pump rotational speed. The value of N is calculated directly from the angular speed ω as

$$N = 30\,\omega/\pi \tag{2}$$

Using the standard methods of Chaudhry (1987) or Wylie and Streeter (1993), the dynamic head of the pump is represented as

$$H_P = H_R(\alpha^2 + \upsilon^2)(a_1 + a_2 \tan^{-1}\alpha/\upsilon) \tag{3}$$

in which a_1 and a_2 are interpolation constants and H_R = rated pump head. However, the particular form of representing $H_P = f(\alpha, \upsilon)$ is often left to the convenience of the modeler, and many other methods have also been suggested (e.g., from simple interpolation to Fourier series).

When the decelerating torque T is assumed to be positive for a pump power failure, the rotational inertia of the pump is described by

$$-T/I = d\omega/dt = f(\omega, \upsilon) \tag{4}$$

where I = rotational inertia of pump impeller, entrained fluid, shaft, and rotor of driver, and t = time. The torque on the pump shaft is defined by $T = \beta T_R$ where T_R = rated torque of pump and β is interpolated from a dimensionless representation of the torque characteristic (Wylie and Streeter 1993). The implicit solution of (4) is thoroughly described by Wylie and Streeter (1993) and Chaudhry (1987). For completeness, the explicit solution of (4), first suggested by Streeter (1969), is reviewed and extended here.

EXPLICIT APPROACH

A second-order explicit approximation of ω is obtained by writing the modified Euler equation in terms of ω. At any time t, the angular speed of the impeller is explicitly defined as

$$\omega_1 = \omega_0 + \Delta t/2[f(\omega_0, \upsilon_0) + f(\omega_1', \upsilon_1')] \tag{5}$$

where subscripts 0 and 1 represent the values of each variable measured at time $t - \Delta t$ and t, respectively, and primed variables represent the predicted quantities at the end of the time step. The predicted relative discharge υ_1' is obtained from a Newton's method solution of the energy equation at time t where the predicted angular speed is defined as

$$\omega_1' = \omega_0 + \Delta t \cdot f(\omega_0, \upsilon_0) \tag{6}$$

in which $\omega_0 = \pi N_R \alpha_0/30$. More specifically, after obtaining the relative speed using $\alpha = 30\,\omega_1'/\pi N_R$, the predicted relative discharge is computed by repeatedly evaluating (1) with different estimates of υ. After each iteration, the estimate of υ is adjusted by a correction factor V_{cor} defined as

$$V_{cor} = -\frac{F_1}{dF_1/d\upsilon} \tag{7}$$

where, from the differentiation of (1) and (3), $dF_1/d\upsilon$ is

$$dF_1/d\upsilon = H_R[2\upsilon(a_1 + a_2 \tan^{-1}\alpha/\upsilon) - a_2\alpha] - dH_\upsilon/d\upsilon + d(H_S - H_D)/d\upsilon \tag{8}$$

The value of υ_1' is determined when the absolute difference between successive estimates of υ is less than a specified tol-

[1]Grad. Student, Dept. of Civ. Engrg., Univ. of Toronto, Toronto, M5S 1A4, Canada.

[2]Prof., Dept. of Civ. Engrg., Univ. of Toronto, Toronto, M5S 1A4, Canada. (Corresponding author.)

Note. Discussion open until August 1, 1998. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on October 21, 1996. This paper is part of the ***Journal of Hydraulic Engineering***, Vol. 124, No. 3, March, 1998. ©ASCE, ISSN 0733-9429/98/0003-0301–0306/$4.00 + $.50 per page. Paper No. 11273.

erance ε. A limit of 10^{-5} was found to be generally suitable. The numerical performance of the explicit approach is illustrated later in this paper.

COMPUTATIONAL ADVANTAGES OF EXPLICIT APPROACH

There are significant programming benefits to be realized from an explicit formulation when modeling pump start up or shut down. Perhaps the best demonstration of these benefits is to examine the computational procedure for complex pumping stations.

Complex Pump Arrangements

Pump stations often comprise more than one pump. Standby units wait to be brought on-line as demand dictates or when maintenance requires that another pump be shut down. The pump boundary condition solver must be able to adjust the system of equations to model the new operating conditions upon pump start-up or shut-down. The explicit approach makes this expansion or contraction of the system both simple and direct.

For example, when one extra pump is added to a system of three pumps in series, an additional torque equation is required. Using the implicit approach, the number of unknowns grows from four to five equations during the transient event, representing the unknown discharge and four speed equations. If the explicit approach is employed and the rotational speed of the additional pump is defined by (5) and (2), the fully transient system must be solved only for the discharge; the system remains 1 × 1. Only one extra term describing the dynamic head of the additional pump is added to the energy equation. Therefore, using the explicit approach, the numerical solution is no more demanding with four pumps than it is with one pump.

If one pump is removed from the system due to valve closure or pump shut down, the dynamic head term of the removed pump is deleted from the energy equation. If the implicit approach is used, the system of equations must also be reduced by one torque equation. Similar computational benefits are realized when modeling variable speed pumping arrangements with the explicit approach.

Variable Speed Pump Arrangements

Systems in which the head and/or flow conditions are constantly changing often employ variable speed pumps. A variable speed pump can operate at or near peak efficiency at several different head/flow combinations. That is, the speed of the pump can be automatically adjusted to meet the head/flow demands. However, this change, if rapid, can result in pressure fluctuations large enough to warrant special protection measures.

Since the network demands are often assumed a priori, the appropriate changes in pump speed to meet the new demands can be calculated and indexed against time prior to beginning the simulation. Thus, the dimensionless pump speed is no longer an unknown. Given that the rotational speed of the pump is known throughout the duration of the simulation, any change in the speed of the pump no longer depends directly on the inertia of the pump impeller and entrained fluid. Therefore, (4) alone no longer governs the speed change of the pump.

Assume, for example, a pumping station comprised of three variable speed centrifugal pumps in series instead of three failing pumps is modeled. Using the implicit approach requires a contraction of the "conventional" system matrix. The variable speed case has only one equation (i.e., the energy equation) and one unknown (i.e., the relative discharge), not the usual four nonlinear equations. If the explicit approach is employed, the only change to the basic algorithm is to replace (2) with the known pump speed. Similarly, if a pump fails during the simulation, the known pump speed is replaced with (2) and the simulation continues as usual. For purely illustrative purposes, the following section describes just such a scenario.

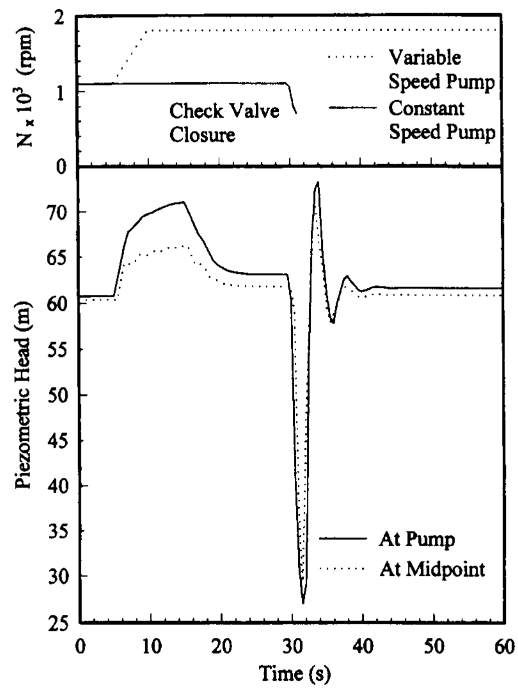

FIG. 1. Variable Speed Pump Undergoing Speed Change and Constant Speed Pump Undergoing Power Failure

Application

Consider a pump station containing two centrifugal pumps in parallel connected to a 1 km long forcemain. The characteristics of both the pumps and the forcemain are found in Chaudhry (1987, p. 110). Assume the first pump is a variable speed unit and undergoes a speed change. The second pump is a constant speed unit and experiences an emergency stop shortly after the first pump reaches its new steady state. To meet increased demand, the speed of the first pump is increased linearly from 1,100 rpm to 1,800 rpm in five seconds beginning at $t = 5$ s. The second pump continues to operate at 1,100 rpm until $t = 30$ s, when an emergency trip is triggered by the operator, and a subsequent closing of the pump check valve occurs at approximately $t = 32$ s. The head trace in Fig. 1 shows the response at the pump (solid line) and at the midpoint (dashed line) of the forcemain due to the combination speed change and power failure. Note that during this transient, the number of system equations need not change because the rotational speed is updated by either the known pump speed (as in the case of the variable speed unit) or the torque equation (as in the case of the failing unit). In fact, all switching and adjusting of the pumping units is performed directly with the explicit approach.

To illustrate the convenience of the explicit approach more clearly and to more convincingly demonstrate its validity, it is compared with the conventional implicit approach in a second example application.

EXAMPLE PUMPING STATION

Consider a pumping station in which three centrifugal pumps are connected in series. But first, for simplicity, reexpress (1) as

$$\Delta H_i = f_{1,i}(\upsilon_i, \alpha_i) \quad (9)$$

and (4) as

$$\Delta\alpha_i = f_{2,i}(\upsilon_i, \alpha_i) \quad (10)$$

where H_i = head at pump i; υ_i = relative discharge of pump i; and α_i = relative speed of pump i. Using the implicit approach, the pump station is defined by the following system of equations;

$$\Delta H_{1\rightarrow 3} = f_{1,1\rightarrow 3}(\upsilon, \alpha_1, \alpha_2, \alpha_3) \quad (11)$$

$$\Delta\alpha_1 = f_{2,1}(\upsilon, \alpha_1) \quad (12)$$

$$\Delta\alpha_2 = f_{2,2}(\upsilon, \alpha_2) \quad (13)$$

$$\Delta\alpha_3 = f_{2,3}(\upsilon, \alpha_3) \quad (14)$$

If the appropriate partial derivatives of (11) through (14) are written, the Newton-Raphson method (e.g., Wylie and Streeter 1993) yields a solution of this 4 × 4 system. However, using the explicit approach, the fully transient system is reduced to a 1 × 1 system defined by

$$\Delta H_{1\rightarrow 3} = f_{1,1\rightarrow 3}(\upsilon) \quad (15)$$

The relative speed of each pump i at time t is

$$\alpha_i(t) = \alpha_i(t - \Delta t) + \Delta t/2\{f_{2,i}[\upsilon(t - \Delta t), \alpha_i(t - \Delta t)] + f_{2,i}[\upsilon'(t), \alpha_i'(t)]\} \quad (16)$$

in which the predicted relative speed at time t is

$$\alpha_i'(t) = \alpha_i(t - \Delta t) + \Delta t \cdot f_{2,i}[\upsilon(t - \Delta t), \alpha_i(t - \Delta t)] \quad (17)$$

In general, the explicit approach consists of two quasi-steady evaluations of the energy equation buried within a modified Euler solution of the torque equation. First, $\alpha'(t)$ is computed by extending the time derivative of $\omega(t - \Delta t)$ to the end of the time step. Then using $\alpha'(t)$, the relative discharge is predicted by a Newton's method solution of the energy equation. Given $\alpha'(t)$ and $\upsilon'(t)$, a corrected value of the relative speed $\alpha(t)$ is determined from the time derivative of $\omega(t)$ and (5). Last, a second Newton's method solution of the energy equation is performed to compute the corrected value of the relative discharge $\upsilon(t)$. Details of the procedure are summarized in pseudocode in Figs. 2 and 3.

The explicit approach is also demonstrated with a pumping station containing two centrifugal pumps in parallel. From (9) and (10), the system is

$$\Delta H_1 = f_{1,1}(\upsilon_1, \alpha_1) \quad (18)$$

$$\Delta H_2 = f_{1,2}(\upsilon_2, \alpha_2) \quad (19)$$

$$\Delta\alpha_1 = f_{2,1}(\upsilon_1, \alpha_1) \quad (20)$$

$$\Delta\alpha_2 = f_{2,2}(\upsilon_2, \alpha_2) \quad (21)$$

Since the discharge through each pump generally differs, two

1. **Initialize Steady State**
2. **While** $t \leq t_{\text{final}}$
   - For $i = 1, 2,$ and 3, solve torque equation at beginning of time step
     - ◇ Compute $\tan^{-1}(\alpha/v)$.
     - ◇ Compute T at beginning of time step using $T = \beta T_R$.
     - ◇ Compute ω at beginning of time step using $\omega_0 = \pi N_R \alpha_0/30$.
     - ◇ Compute $d\omega/dt$ at beginning of time step using (4).
     - ◇ Compute first estimate of ω at end of time step using (6).
     - ◇ Compute first estimate of N at end of time step using (2).
     - ◇ Compute first estimate of α at end of time step using $\alpha = N/N_R$.
   - Using Newton's method (see Fig. 3), solve energy equation for first estimate of v at end of time step. Begin Newton's method with $v = v_0 + \Delta v$.
   - For $i = 1, 2,$ and 3, solve torque equation at end of time step
     - ◇ Compute $\tan^{-1}(\alpha/v)$.
     - ◇ Compute T at end of time step using $T = \beta T_R$.
     - ◇ Compute $d\omega/dt$ at end of time step using (4).
     - ◇ Compute second estimate of ω at end of time step using (5).
     - ◇ Compute second estimate of N at end of time step using (2).
     - ◇ Compute second estimate of α at end of time step using $\alpha = N/N_R$.
   - Using Newton's method (see Fig. 3), solve energy equation for second estimate of v at end of time step. Begin Newton's method with $v = v_0 + \Delta v$.
   - Set $\Delta v = v - v_0$.
   - Set $t = t + \Delta t$.
3. **STOP**

FIG. 2. Pseudocode for Explicit Approach

1. **Initialize all tolerances**

2. **While** $j \leq 3$

   - Compute $\tan^{-1}(\alpha/v)$.
   - Compute energy equation and its derivative with respect to v using (1) and (8).
   - For $i = 1, 2,$ and 3,
     - ◇ Compute correction to v estimate V_{cor} using (7).
     - ◇ Compute new estimate of v, $v = v - V_{cor}$.
     - ◇ If $|V_{cor}| \leq \varepsilon$, $j = j + 1$.

3. **RETURN**

FIG. 3. Pseudocode for Newton's Method Solution of Energy Equation

energy equations are written to fully describe the head change associated with each pump. When the appropriate partial differentials are computed, the resulting 4 × 4 system is again evaluated using the Newton-Raphson procedure. Alternatively, the explicit approach reduces the system of equations to

$$\Delta H_1 = f_{1,1}(\upsilon_1) \tag{22}$$

$$\Delta H_2 = f_{1,2}(\upsilon_2) \tag{23}$$

The relative speed of each pump is again defined by (16), and therefore (22) and (23) can be solved by Newton's method for the relative discharge of each pump.

Of course, while this kind of example may clearly indicate the convenience of the explicit approach, it does nothing to indicate its accuracy. This is discussed next.

MODEL VERIFICATION

The intent of the explicit approach is to improve the management of transient simulation models as opposed to improving their ability to predict measured results from a field test. Therefore, the most suitable verification is to compare the performance of both implicit and explicit approaches directly with an "exact" solution of the pump boundary condition. An "exact" solution means an accurate numerical solution associated with a very fine temporal discretization.

Once again, consider a pumping station containing two centrifugal pumps in parallel connected to a 1 km long forcemain. As before, the characteristics of both the pumps and the forcemain can be found in Chaudhry (1987, p. 110). The head trace in Fig. 4(a) shows the response at the pump station due to a power failure as computed by the implicit and explicit approaches. Fig. 4(b) shows the deviation of the implicit approach from the exact solution as well as the deviation of the explicit approach from the exact solution. The exact solution is obtained using the implicit approach and a time step of 1/64 s. As Fig. 4(b) illustrates, both the implicit and explicit approaches differ from the exact solution by as much as 1.1 m when the time step of both approaches is 1/4 s. Although the deviation of the explicit approach from the exact solution is greater than the deviation of the implicit approach, the explicit approach executes more rapidly than does the implicit approach. It is worth pausing here to discuss the issue of accuracy.

The accuracy of a numerical method is measured by its deviation from a standard or accepted solution, previously referred to as an "exact" solution. For a given step size, one numerical method is deemed more accurate than another if it deviates less from the exact solution. However, ignored in this criterion is the computational effort required to obtain each

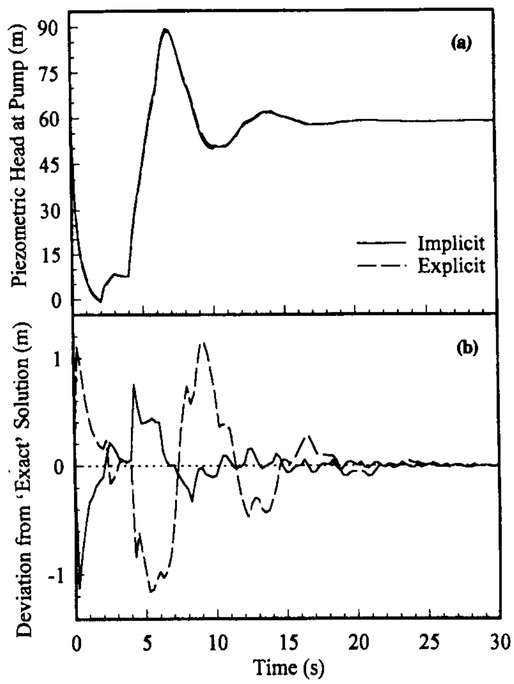

FIG. 4. Deviation of Implicit (Δt = 1/4 s) and Explicit (Δt = 1/4 s) Approaches from "Exact" Solution

solution. For example, compare the modified Euler method requiring two functional evaluations per time step with the Euler method requiring only one functional evaluation per time step. If the modified Euler method is superior in terms of accuracy, it should deviate less from the standard solution with step size Δt than the Euler method with a step size $\Delta t/2$, because the modified Euler method requires twice as many evaluations per step (Burden and Faires 1985). The point being made here is that the computational expense or execution time should not be blindly ignored when comparing the accuracy of competing numerical methods. The question is always: Accuracy at what price?

Returning to the torque equation solutions, remember that the explicit approach executes more rapidly than does the implicit approach; therefore, a responsible comparison of accuracy should include consideration of computational expense. For Δt = 1/6 s, the explicit approach was found to require approximately the same computational resources as the implicit approach when Δt = 1/4 s. As Fig. 5 shows, the deviation of both approaches from the exact solution is generally equivalent when the computational expense is generally equivalent. The intent here is not to show that the implicit approach is