

# A Systematic Exploration of Uncertainty and Convergence of Inverse Transient Calibration for WDSs

B. S. Jung<sup>1</sup> and B. W. Karney<sup>2</sup>

<sup>1</sup>PhD Candidate, Department of Civil Engineering, University of Toronto, 35 St. George St., Toronto, ON, M5S 1A4, Canada; email jung@ecf.utoronto.ca

<sup>2</sup>Professor, Department of Civil Engineering, University of Toronto, 35 St. George St., Toronto, ON, M5S 1A4, Canada; email: karney@ecf.utoronto.ca

## *Abstract*

Despite over ten years of research into ITC techniques for water distribution systems, many problems remain. One reason for these difficulties is that real water distribution systems invariably have many other uncertainties in addition to the leakage rates and friction factors that are conventionally considered as unknowns. For example, properties such as pipe diameter, wave speed, the possible presence of air, the value of the water demand at the time of the tests, and uncertain measurement accuracy, all add to the complexity and difficulty of obtaining a reliable calibration. The current paper investigates quantitatively how several of these uncertainties deteriorate system calibration, and thus the paper generally considers the necessity of a systematic calibration approach to explicitly include these additional uncertainties during the ITC process. To this end, two evolutionary optimizations, namely Genetic Algorithms and Particle Swarm Optimization, are compared and contrasted during the ITC iterations. The advantage of the evolutionary algorithms is that they help the search to escape from poor local optima in multifaceted and complicate problems and thus to locate a good global (or near-global) optimum. However, even these approaches can often be expected to converge poorly when the full scale of the field problem is reflected in the search space.

## *Introduction*

Distribution networks are an essential part of all water supply systems. The construction and maintenance of these pipelines system cost many millions of dollars every year. One of the challenging and difficult (but important) issues for water distribution systems is the accurate estimation of the pipe internal roughness (or friction factor) and the detection of both normal demands and leakage in the pipeline system.

Finding leaks and calibrating the friction factor is, at least, theoretically possible using inverse methods where the results of measurements are known but parameters of physical system are unknown. In essence, inverse methods seek to determine the physical data that, when input into a simulation model, will yield a reasonable match between measured and predicted pressures and flows in the pipe network. Recently, a calibration method called “Inverse Transient Calibration” (ITC) has been widely investigated and advocated (Liggett and Chen, 1994; Nash and Karney, 1999; Vitkovsky et al., 2000;

Jung and Karney, 2004). The approach in this case is to create a transient condition and to use high frequency pressure transducers to record the system's response. Next, an optimization model is used to change system parameters to bring the predicted and measured responses into reasonable agreement.

Many different optimization models and approaches have been developed for inverse transient calibration. Gradient methods like the Levenberg-Marquardt method were initially most widely used (Liggett and Chen, 1994; Nash and Karney, 1999). The main advantage of the gradient type optimization is their computational efficiency. However, several researchers noted that gradient type methods may fail to converge, due to instability problems, or may converge to a local rather than global optimum. Recently, Genetic Algorithms (GA) have been popular in water distribution optimization (Goldberg, 1989; Simpson et al., 1994; Dandy et al., 1996; Jung and Karney, 2005). The GAs provide solutions to many hard optimization problems that are difficult to cope with using the traditional gradient type methods. The main advantage of these evolutionary algorithms is their ability to find the global optimum. A population of potential solutions in an evolutionary procedure explores the search space simultaneously, exchanges information among the trials and uses only function values (not derivatives) of the objective function. Vitkovsky et al. (2000) performed GA optimization in conjunction with the inverse transient method to detect leaks and frictions in WDS.

Despite many years of research into ITC techniques for water distribution systems, many significant challenges remain for field application. Even though model-based methods are excellent for numerical calibration tests (Liggett and Chen, 1994; Vitkovsky et al., 2000), these methods haven't been widely applied to field experiments. One of the main reasons is that the existing transient models do not yet represent the full and complex behavior of a system accurately and many complications, like unsteady friction factors, remain as unresolved issues. Other reasons for these difficulties in real water distribution systems are the many uncertainties in addition to the leakage rates and friction factors that are conventionally considered as unknowns. For example, properties such as pipe diameter, wave speed, the possible presence of air, the value of the water demand at the time of the tests, and uncertain measurement accuracy, all add to the complexity and difficulty of obtaining a reliable calibration.

This paper explores the impact of several of these uncertainties, including pipe diameter, water demand and wave speed, in water distribution system calibration and examines systemically how these uncertainties deteriorate or complicate convergence. As a consequence, a systematic ITC approach to include these uncertainties in addition to leak and friction factor is considered and the corresponding case studies are explored. Two evolutionary optimizations, Genetic Algorithms and Particle Swarm Optimization, are compared and contrasted during the ITC iterations. The advantage of the evolutionary algorithms is that they help the search to escape from poor local optima in multifaceted and complicated problems and thus help to locate a good global (or near-global) optimum; however, even these approaches turn out to converge poorly when the full scale of the field problem is reflected in the search space.

### ***Inverse Transient Calibration***

Leak detection and system parameter calibration can potentially be achieved by determining the data that when input into a hydraulic simulation computer model will yield a reasonable match between measured and predicted pressures and flows in the network. In order to calibrate the water distribution

system for parameters describing leakage and friction factors, steady state approaches have been attempted (Boulos and Wood, 1990; Pudar and Liggett, 1992); but the distinct and important drawback of steady state analysis is how to acquire sufficient independent data for pipeline system calibration. Chen and Liggett (1994) introduced inverse transient analysis, not only for integrated analysis, but also for integrated calibration with transient data and models. In essence, this approach creates a transient condition and then uses high frequency pressure transducers to record to the system's response. Next an optimization model, like a Levenberg-Marquardt algorithm or other search approach, is used to bring the predicted and measured responses into reasonable agreement. This procedure has the capacity to collect and compile many orders of magnitude more data than is economically available with a steady state approach. However, the gradient-based optimizations like Levenberg-Marquardt algorithm, despite their potentially fast convergence in simple systems, require gradient data (e.g., Jacobian and Hessian matrices), which are sometimes problematic in complicated networks. Another crucial drawback of the gradient-based optimization is that their characteristic may cause the solution to converge to a local optimum, rather than a global one, or fail to converge at all.

### *Uncertainties of Inverse Calibration*

The primary objective of a simulation is to reproduce the behavior of a real system and its spatial and dynamic characteristics in a useful way. To accomplish this goal, data are supplied that depict the physical characteristics of the system, the loads placed on the system, and the boundary conditions in effect (Walski et al., 2001). Even if all of the data gathered describing the model match the real system, the calculated pressures and flows always have some discrepancies with the observed ones. One of the main reasons arises from mathematical assumptions associated with achieving computational tractability in the simulation model and associated technical difficulties in representing the real system. Classic water hammer theory based on the assumptions of linear elastic behaviour of pipe-walls and quasi-steady state friction losses is used to predict the maximum and minimum pressure surges; this is relatively accurate for simulating hydraulic transients in metal pipe, but it is considerably less precise for plastic pipes, particularly when the surge is generated by rapid changes in flow conditions. This unsteady friction characterization is a challenging question and research is still ongoing (e.g., Brunone et al., 1991; Vitkovsky, 2001).

Another critical reason for the problems of ITC, especially for real field tests, is the uncertain representation of the real system. Traditionally, ITC is accomplished by only adjusting internal pipe friction values and nodal leakages until an agreement between observed and computed pressures and flows are obtained. Generally speaking, the basis for this claim is that the system characteristics like pipe diameters, water demands, wave speeds, pressures and flows are directly measured; on the other hand, pipe friction values and nodal demands are typically estimated. Hence, there is room for adjustment in these values in order to improve the estimation. However, numerous factors can contribute to disagreement between model and field observation. Any input data that is uncertain, but easily ignored, are candidates for adjustment during calibration to obtain reasonable agreement between model-predicted behavior and actual field behavior.

Determining pipe diameter is not as straightforward as it might seem. The internal diameter of pipe may differ from the nominal diameter which is all too commonly used in modeling. To add to the confusion, the internal diameter may change over time as corrosion, tuberculation, and scaling occur

within the pipe. Corrosion and tuberculation are related in iron pipes. As corrosion reactions occur on the inner surface of the pipe, the reaction by-products expand to form an uneven pattern of lumps (or tubercles) in a process which is known as tuberculation. Scaling is a chemical deposition process that forms a material build-up along the pipe walls due to chemical conditions in the water. Scaling can actually be used to control corrosion, but when it occurs in an uncontrolled manner it can significantly reduce the internal diameter of the pipe (Walski et al., 2001). The reduced pipe diameter has a great influence on the head loss through a pipe. A 10 percent decrease in the pipe diameter will increase head loss by nearly 40 percent (Walski et al., 2001). Steady state hydraulic calibration models generally employ a resistance coefficient, instead of adjusting both the friction coefficient and diameter. By adjusting friction coefficients in calibration but leaving diameter at assumed known values, the number of variables is minimized by half and the calibration process is simplified. However, this mathematical simplification does not likely represent the physics of water distribution systems. Moreover, it would be even more problematic during transient calibration because pipe diameter is strongly linked through velocity to system dynamics.

Another uncertainty in water distribution system is water demand. Water usage in municipal water distribution systems is inherently unsteady due to continuously varying demands. In order for a transient simulation to accurately reflect the dynamics of the real system, these demand fluctuations should be incorporated into the model; but, surprisingly, little is known about exact values of instantaneous demand in complex real pipe systems. Also, there is a spatial problem that the water distribution modeling equations are based on the simplifying assumption that water is withdrawn at a junction node. In reality, however, water usage tends to occur along the entire length of pipe. Spatially redistributing water usages that occur along a length of pipe to the junction nodes in the model is widely used and known as "demand allocation". The demand allocation process is a possible source of error that should be considered when calibrating a model. It is conceivable that a model could incorporate all of the locations where water is withdrawn from the system by placing junction nodes where the service lines are connected to the water main. This approach, however, would significantly increase the number of pipes required in the model, thereby increasing its complexity and the data requirements. McInnis and Karney (1995) present a formulation to permit system demand as a distributed pipe flux in transient model.

Wave speed is another challenging uncertainty in a pipeline. Wave speed is a function of many fluid and pipe properties (pipe diameter, thickness and material; water density, elasticity, temperature, air and solids content; pipe restraint conditions, etc.). Some of these conditions can be accurately assessed, but many are not well defined and difficult to estimate. For example, accurate measurement of the air content dispersed in fluid is difficult; however, even a tiny amount of gas throughout a liquid greatly reduces the propagation velocity of a pressure wave in a pipeline (Wylie and Streeter, 1993).

In this paper, three uncertainties: pipe diameter, water demand and wave speed are considered through several case studies to examine systemically how these uncertainties deteriorate system calibration. In addition, an ITC approach including these uncertainties, in addition to leak and friction factor, is explored. One of the goals is to assess the limits of the ITC approach when facing a significant uncertainty; that is, to determine when ITC is likely to fail.

## *Evolutionary Computation for Optimization*

Evolutionary Computation (EC) is an optimization system inspired by biological evolution and adaptation. It aims at understanding such natural computational systems and developing more robust and efficient algorithms for solving complex real-world optimization problems. The specific problems dealt with by such computational systems are usually highly nonlinear and often contain inaccurate and noisy data (Back et al., 1997).

The most well known EC paradigm is the Genetic Algorithm (GA), created by John Holland and made popular at least for engineers by Goldberg (1989). The GA approach is used widely, especially in engineering and industrial applications. In the GA approach, the population is binary encoded and genetic operators, inspired to mimic DNA evolutionary procedures, are applied to the population in order to stimulate the system to evolve, and thus to explore the search space efficiently.

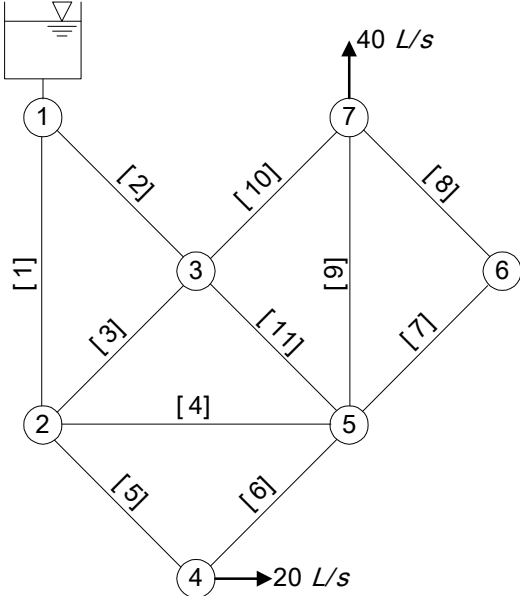
Recently, a new EC field has arisen, based on another population search procedure, called Swarm Intelligence (SI). SI argues that intelligent cognition derives from the interaction of individuals in a social environment and that the main ideas of sociocognition can be effectively applied to develop stable and efficient algorithms for optimization tasks. One of SI techniques, called Particle Swarm Optimization (PSO), has been developed by Kennedy and Eberhart (1995) to simulate the movement of a flock of birds searching for food. PSO has been used mainly for continuous optimization in which the population of potential solutions is called a “swarm.” In the computer version of this search, the global exchange of information among all individuals, which are called “particles”, takes place and each particle can profit from the discoveries of the rest of the swarm. Many variants of the PSO technique have been developed. In this paper, a version of the algorithm derived by adding an inertial weight to the original PSO dynamics has been used (Shi and Eberhart, 1998; Jung and Karney, 2004; Jung and Karney, 2005).

## *Case Studies*

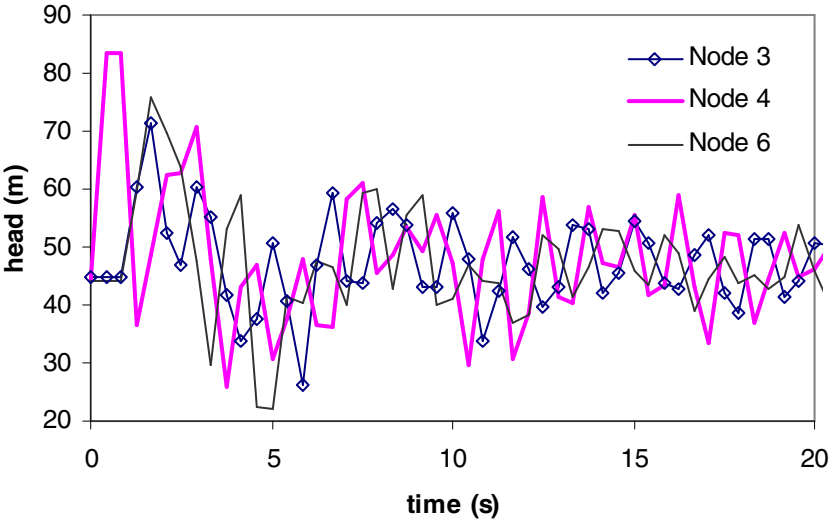
A case study, based on a network introduced by Pudar and Liggett (1992) and applied to several other approaches (Liggett and Chen, 1994; Vitkovsky et al., 2000), is presented to illustrate a more challenging calibration application. The network system shown in Fig. 1 comprises one reservoir, 7 nodes and 11 pipes. Water is drawn from the higher reservoir (50 m) to the downstream network and supplied to two external demands (at nodes 4 and 7 with flows of 20 L/s and 40 L/s, respectively). The pipe diameter, length and Darcy-Weisbach friction factors for all pipes are given as 0.2 m, 500 m and 0.02, respectively. The pipe material is set as steel with a wave speed of 1200 m/s. One computational reach is selected for each pipe so the time step is 5/12 s. All nodes except the reservoir are at datum.

To initiate transient conditions for the ITC procedure, a simple transient event caused by an instantaneous valve closure at node 4 is assumed. The leak point is located at node 2 and the leak area is 0.001 m<sup>2</sup>. Fig. 2 shows the transient head traces of node 3, 4 and 6 due to the rapid valve closure at node 4. The possible leakage nodes are selected as all “internal” nodes (4 unknowns), that is, all nodes except the locations of reservoir and the demand locations. The permitted range of leak area is 0 to 0.005 m<sup>2</sup>. Pipe friction factors (11 unknowns) are chosen for all pipes within the range of 0.012 to 0.04. The number and location of measurement sites are selected for each case study differently; the length

of record for the measurement sites is fixed as 40 s, so each measurement site provides 97  $(40/5/12+1)$  values of the pressure head.



**Fig. 1 Network System of 7 nodes and 11 pipes**

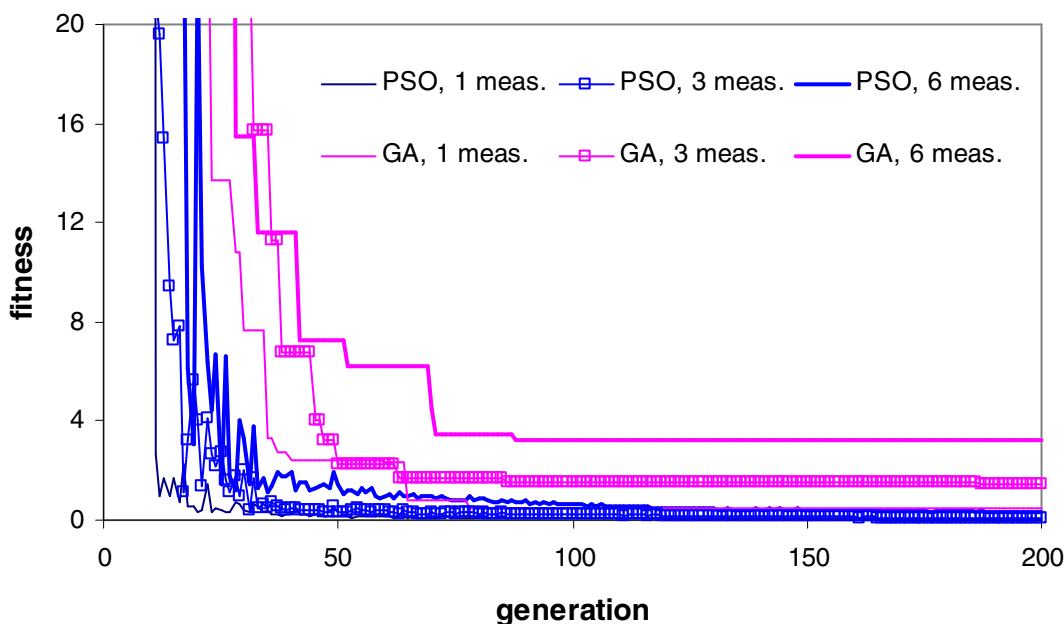


**Fig. 2 Head traces: measurement data for forward analysis**

Two optimization methods, GA and PSO, are considered for controlling the search during the inverse calculation. In the GA, the probabilities of mutation and (uniform) crossover are 0.02 and 0.5, respectively. Tournament selection and elitism (in which the best individual is copied to the next generation) are selected. In the PSO, a linearly decreasing inertia weight is used which starts at 0.9 and ends at 0.4, with  $c_1 = 2$  and  $c_2 = 2$  (Shi and Eberhart, 1998). The maximum velocities for leakage and friction factor are 0.0005 and 0.003, which are approximately 10% of search ranges. The population size and generation (iteration) number for both optimizations are selected depending on specific case studies.

**ITC for leakage and friction factor.** Traditional ITC approaches that only consider leakage and friction factor are first explored. The population size and number of generations (iterations) for both optimizations are 200 and 200, respectively. For the inverse calculations, three different measurement combinations – one site (node 6), three sites (node 3, 4 and 6) and six sites (node 2 to 7) – are selected to see how this data influences the ITC results. The objective function of GA and PSO is to minimize the square sum of the difference between measured head trace and the predicted values during the inverse calculation.

Fig. 3 shows the search results for finding the leakage for 4 possible leakage nodes and friction factors for all pipes. Both optimizations show, regardless of the number of measurement sites, fast initial convergence during the first 50 generations; PSO exhibits a faster and more precise convergence than does GA. Table 1 presents the calibration results using GA and PSO with different combinations of the measurement sites. Fig. 3 and Table 1 show that fewer measurement sites, in both optimizations, leads to better fitness values and faster convergence, but less accurate calibration results. Fewer measurements allow quick convergence to the wrong answer. Also, Table 1 shows both optimizations provide accurate calibration results for leakages,



**Fig. 3 Evolution procedure for leakage and friction factor**

**Table 1 Leakage and Friction Factor calibration with GA and PSO**

	Node /Pipe	Real	Meas. 1		Meas. 3		Meas. 6	
			GA	PSO	GA	PSO	GA	PSO
Possible leakage node ( $10^{-3} \text{ m}^2$ )	2	1	1.02	1.00	1.02	1.00	1.02	1.00
	3	0	0.00	0.00	0.00	0.00	0.00	0.00
	5	0	0.00	0.00	0.00	0.00	0.00	0.00
	6	0	0.00	0.00	0.00	0.00	0.00	0.00
Unknown friction factor ( $10^{-2}$ )	1	2	1.77	2.28	1.88	2.06	2.13	1.95
	2	2	2.06	1.77	2.04	1.94	1.80	2.04
	3	2	1.64	3.24	1.47	2.79	2.39	1.93
	4	2	2.70	3.00	1.42	2.26	3.58	1.52
	5	2	3.38	4.00	3.45	4.00	1.33	4.00
	6	2	2.32	2.97	3.76	4.00	2.90	1.20
	7	2	3.27	4.00	3.89	3.46	2.68	2.73
	8	2	1.55	3.11	3.25	2.50	2.19	2.93
	9	2	1.53	1.98	1.46	2.19	1.88	1.85
	10	2	1.84	1.20	1.46	1.20	1.53	1.77
11	2	1.86	2.77	3.29	3.22	2.21	2.61	
Fitness		-	0.453	0.0324	1.46	0.0341	3.23	0.346

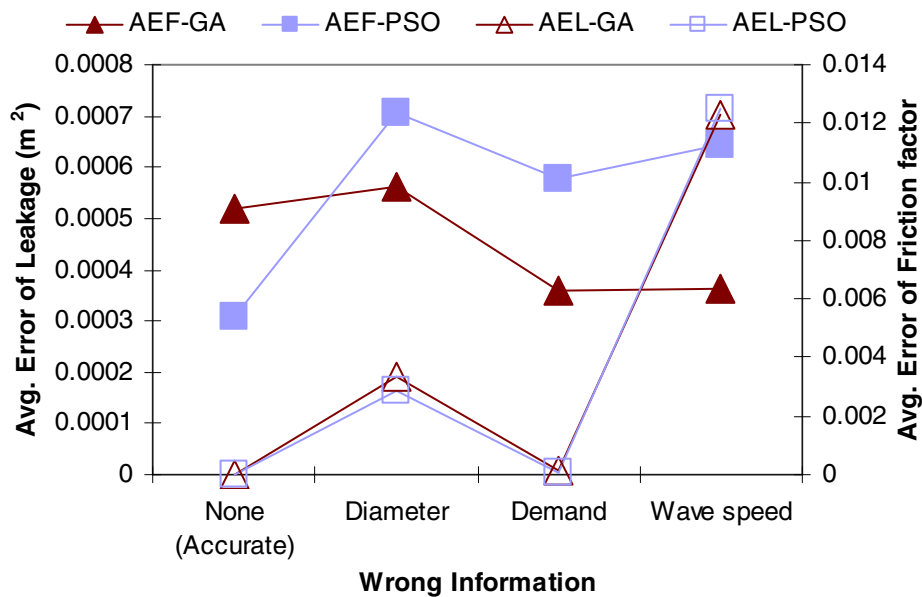
but show less precision for the friction factors. Jung and Karney (2004) also found that the inverse calibration of friction factor is less accurate than that of leakage in their numerical experiments. Vitkovsky et al. (2003) describe the reason: the sensitivities with respect to the leaks are typically three orders of magnitude greater than those with respect to the friction factors. The mismatch in sensitivities, therefore, causes the optimization to occur for the leak parameter rather than for both parameter types.

**ITC with wrong system information.** The same ITC calculations for leakages and friction factors are now performed again, but now with three new sources of uncertainty: pipe diameter, water demand and wave speed. These terms provide a greater degree of realism but complicate the convergence problem. The parameters for both optimizations are same as those in the previous case but measurement sites are fixed with three nodes (node 3, 4 and 6).

Different pipe diameters and wave speeds in the system create uneven travel times in constructing characteristic grids. The smallest Courant number in the system can be adjusted to unity by dividing the pipe into smaller computational units. The process of discretization is repeated until the smallest Courant number exceeds 0.75. After the discretization, a linear timeline interpolation (Jung and Karney, 2005) is used to obtain head and flow at a grid point in the characteristic mesh for the uneven computational units. This rediscretization problem arises whenever wave speed or pipe lengths are uncertain.

First, a 20% error in the diameter of pipe 6 (0.2 m to 0.16 m) is introduced and its impact on convergence is explored. Second, incorrect information for the demand at node 7 is explored. For simplicity, the water demand is assumed constant with time but a 20% error (40 L/s to 48 L/s) is introduced. Third, the wrong information for the pipe material is considered. A plastic pipe, instead of steel, at pipe 6 is assumed, so the corresponding wave speed is taken as 300 m/s, not 1200 m/s as it was in the forward calculation. The squared sum of the difference between measured and calculated data, with determined leakages and friction factors using wrong information of diameter, demand and wave speed are 10600, 230 and 20200, respectively. Hence, at least in this case, the wave speed is most influential parameter among the three deteriorating ITC in this case studies.

Fig. 4 shows the average error between real and calibrated leakages and frictions using incorrect base information. The fitness values of GA are similar with those of PSO and the fitness results using the wrong information of diameter, demand and wave speed are 5300, 3.1 and 17000, respectively. Obviously, the wrong information of system parameter contaminates the system calibration so the calibrated results are, not surprisingly, much poorer than those of the case without wrong system information. Due to the varying influence of each term, water demand errors are less influential than pipe diameter and wave speed. Similarly to the previous case results, the case studies using the wrong water demand show the leak calibration is relatively accurate but the friction factor calibration was less so. However, the case of wrong pipe diameter provides poor calibration results for leakages as well as friction factors. Because pipe diameter has a more direct influence on the system dynamics, in practice a small amount of diameter discrepancy can greatly contaminate the calibration results using transient signals. The wrong information for wave speed in the three case studies gives rise to the worst calibration result. Fig. 4 clearly demonstrates that neither optimization approach can find the real

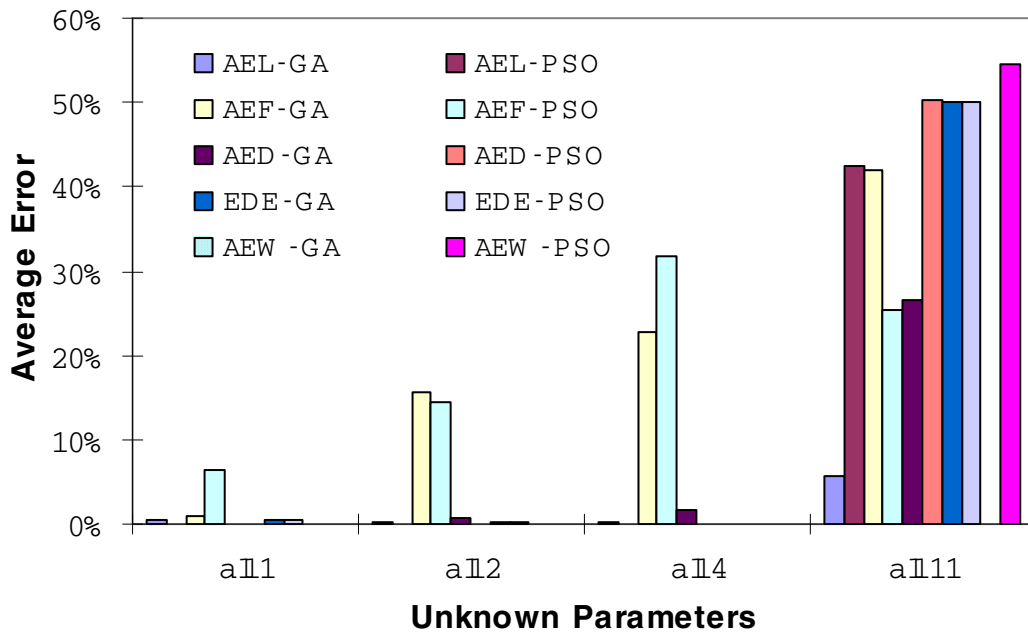


**Fig. 4 Average Errors of Leakage and Friction Factor (AEF: Average Error of Friction factor, AEL: Average Error of Leakage)**

friction factors for all cases, nor even the leakages, when the wrong wave speed is used. The reason for the poor calibration results with the wave speed case is that the wave speed at pipe 6 strongly affects not only the magnitude of the transient response, but also the phase timing of the dynamic behavior. The distorted transient phase causes a significant mismatch during the ITC.

**ITC with all uncertainties.** Since the case studies show that only a small amount of wrong information about the system parameters causes a high contamination of the ITC results, an obvious remedy is to treat all parameters that are uncertain as unknowns. Hence, calibration with all parameters is potentially desirable for ITC, especially for field applications.

The current case study considers all parameters as calibration candidates: 4 possible leakages at nodes 2, 3, 5 and 6; 11 unknown friction factors at all pipes; 11 unknown pipe diameters at all pipes; an unknown water demand at node 7; 11 unknown wave speeds at all pipes. However, an increased number of unknowns and different sensitivities of system parameters are crucially challenging problem in both PSO and GA optimizations. The results of the case studies with inaccurate system information indicate that wave speed strongly affects the magnitude and phase of timing of transient response so the wave speed is often one of the most sensitive parameters. Preliminary calibration tests are explored, specially considering the sensitivity of wave speed and find that the wave speeds, at least in this cast study, affect more strongly the inverse calibration procedure than the other parameters, so this uncertainty tends to spoil the calibration of the others. Therefore, a more simplified binary range of wave speeds, 300 m/s or 1200 m/s are considered in the case studies. The two values of wave speed correspond with the different pipe materials such as plastic and steel.



**Fig. 5 Average errors with different number of parameters (AED: Average Error of Diameter, EDE: Error of Demand, AEW: Average Error of Wave speed)**

Due to the complexity of the optimization, the calibration starts with one unknown value of all parameters and progressively increases the number of the unknowns to two, four and, finally, all (eleven) variable of the parameters. Due to the increase of the number of unknowns, the population size and generation (iteration) number for both optimizations are set as 500 and 200, respectively. For inverse calculation, three measurement sites at node 3, 4 and 6 are selected. The ranges of leak area and friction factor are same as for the previous case studies. The ranges of pipe diameter and water demand are 100 mm to 200 mm and 20 L/s to 60 L/s, respectively. For the PSO approach, the maximum velocities of pipe diameter and water demand are set as 10 mm and 4 L/s, respectively.

Fig. 5 shows the average relative errors between real and calibrated five parameters. For comparison, each error is divided by its search range to show its relative error. Both optimizations in Fig. 5 show accurate calibration for first three cases (all 1, 2 and 4); however, they do not perform well for the case of all 11 parameters. The reason for the poor convergence is likely the different sensitivities of the parameters as well as the increased number of unknowns. Certainly, the friction factor is always the least accurate due to its low sensitivity.

### ***Conclusion***

Although Inverse Transient Calibration (ITC) is a mathematically elegant method to find system parameters using system responses, many challenging problems remain when the full complexity of a field application is explored. Complexity and uncertainty in water distribution systems substantially deteriorates the conventional ITC of leakage and friction factor. These difficulties in truly complex problems, at least, partly limit ITC to numerical and laboratorial experiments or well defined field systems. This paper investigates uncertainties in water distribution system and quantitatively discovers how the uncertainties influence system calibration. Pipe diameter, water demand and wave speed are considered in this paper and a systematic calibration approach to include these additional uncertainties during the ITC process is presented.

As optimization methods, Genetic Algorithm and Particle Swarm Optimization are applied to minimize the difference between the measure head trace and the predicted one during the inverse calculation. In the case studies with some incorrect information, both optimizations provide relatively accurate calibration for leakages but poorer values for friction factors. The mismatch in sensitivities creates a difficult inverse problem for low sensitive parameters, especially in the systematic calibration to include all uncertainties. Hence, these varying sensitivities should receive special attention in the future along with the many other practical challenges of ITC.

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